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SAPPHIRE WHISPERING GALLERY RESONATORS WITH MODIFIED TEMPERATURE COEFFICIENT OF FREQUENCY

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Abstract - **This paper deals with sapphire "whispering gallery" resonators (SWGR) and studies particularly a way to modify their temperature coefficient of frequency without using of dielectric or mechanical compensation. Instead, the proposed method implies special shaping of a disk SWGR made from a uniform monocrystal as described in the preceding paper [1].**
Keywords **– Microwave resonator, sapphire, whispering gallery mode, temperature coefficient of frequency**

I. INTRODUCTION

Last year it was shown that temperature coefficient of frequency (TCF) of the langatate disk resonant WGE-modes can be tuned at a desirable value, frequency-temperature compensation including, if the disk has double-sided wedge-shaped grooves (V-type grooves) made on the disk butt-ends in vicinity of a radial electric field maximum. Physical basis of this method, as explained in [1], is associated with features of electromagnetic field configuration inside the “whispering gallery” resonator (WGR) at different modes and specific properties of dielectric permittivity of the used crystal.

This paper repeats the same analysis for sapphire "whispering gallery" resonators (SWGRs) that are especially important in practice as providing the highest Q-factor. Unfortunately, SWGR has rather high TCF out of the helium temperature range. In result, it is easier to build a SWGR based super-low phase noise microwave source than provide its good long-term frequency stability.

The paper contents results of 3D finite element (FE) electrodynamic SWGR modeling. We investigate TCF and resonant frequency dependences on temperature (*T*), disk format, mode polarization, azimuthal indices and groove parameters. It is unreasonable, of course, to expect here drastic TCF changes like those for the langatate resonators so far as both components of a sapphire dielectric permittivity tensor (ε_z in plane parallel to *z*-axis and ε_r in plane perpendicular to *z*-axis) as against to langatate demonstrate similar behavior in respect of temperature [2, 3]. Nevertheless, we hope to derive recommendations that could be used when one needs to tune TCF at some particular value. One possible application might be connected with [4] where it was proposed to get better stability using a difference frequency between two SWGR modes of orthogonal polarization [4]. This technique works if TCF values of the two modes are matched precisely. No doubt, a simple method allowing to modify TCF might be useful here.

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II. METHODOLOGY

Our method [1] explores the known fact that due to crystal anisotropy the sapphire dielectric permittivity along an optical axis, ε_z, and that one in a perpendicular direction, ε_r, changes with different rates when temperature varies. The main effect arises due to suitable auxiliary local heterogeneity on the disk surface creates artificial sections of electric line of forces in parallel with the z-axis. The particular realization in [1] looks like a double-sided wedge-shaped (V-type) groove made on disk butt-ends in vicinity of a radial electric field maximum (Fig. 1). Choosing the groove width and depth, the TCF of a particular SWGR mode can be tuned within some limits.

The simulation is performed using the same 3D FE software as in [1]. The computer configuration includes a modern CPU, a RAID-controller and 1.5 Gb of DDR-type memory.

Accurate determination of the resonant frequencies requires a precise knowledge of the sapphire ε_z and ε_r versus temperature functions. The standard polynomial ε_r(T) representation like in [3] and more accurate ε_z(T) description from ref. 10 of [2] are used in analysis.

To simplify calculations we neglect influence of sapphire thermal expansion coefficient (TCL) difference along two characteristic axes. It results in the averaged TCL ≈ -5.43 ppm/degree at $T = 300$K.

III. SIMULATION RESULTS

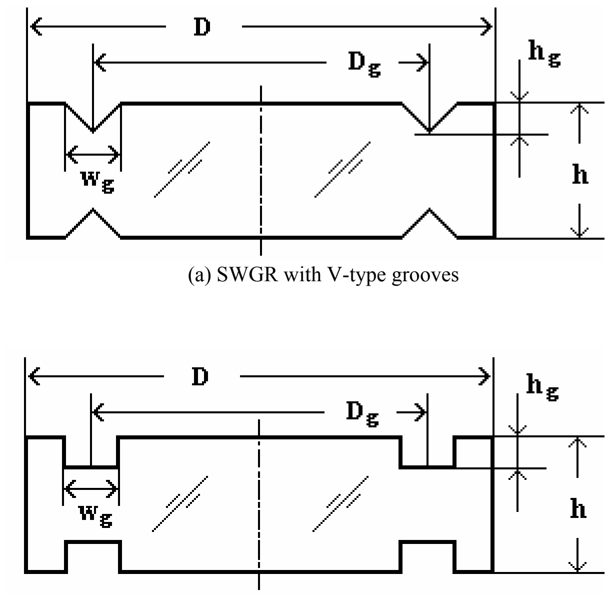
A. V-type Grooves

The SWGR view in this case is shown in Fig. 1a. All simulations are made for the SWGR having “disk format”

$$\Phi = \frac{h}{2D} = 1.97\,. \qquad (1)$$

Here *D* – a diameter, *h* – a height of the disk. The grooves are centered with respect to maximum of a radial electric field of the regular WGE_{711} mode. The radius of a groove center, $R_g = D_g/2$, is 14 mm when $D = 37$, $h = 9.4$ mm.

Under these conditions the groove decreases the |TCF| of WGE-modes the more the deeper the groove (Fig. 2). In the case of groove depth equal 3.7 mm the TCF variation is about 4% and 5% for WGE_{711} and WGE_{611} modes accordingly. The groove of 4.07 mm in depth provides ≈8% TCF change for WGE_{611} mode.



(a) SWGR with V-type grooves

(b) SWGR with rectangular grooves

Fig. 1. Modified SWGR profiles.

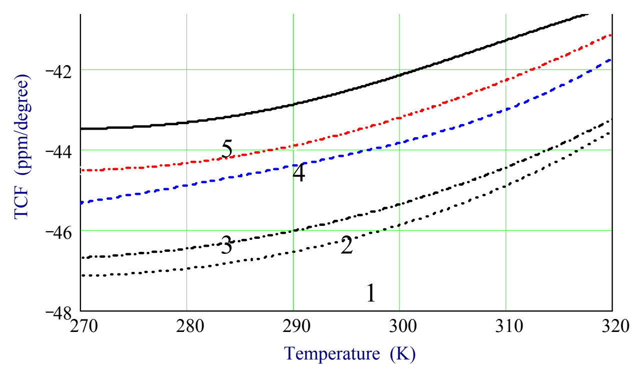


Fig. 2. TCF vs. temperature plots for a regular SWGR (1, 2) and the SWGR with V-type grooves (3-5). Modes: curves 1, 3 – WGE_{711}; 2, 4, 5 – WGE_{611}. Groove depth: curves 3, 4 – 3.7 mm; 5 – 4.47 mm. Groove width = 2 mm.

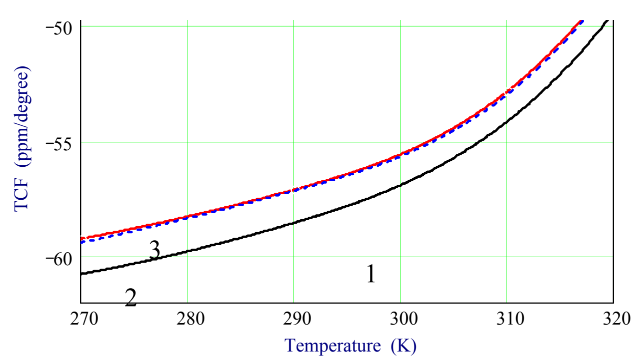


Fig. 3. TCF vs. temperature plots for a WGH_{811}-mode. Curve 1 – a regular sapphire disk; 2, 3 – SWGR with V-type grooves.
Groove depth: curve 2 – 3.7 mm, 3 – 4.47 mm. Groove width = 2 mm.

Analogous plots in Fig. 3 for WGH_{811} mode show sufficiently weaker groove impact. It is a consequence of chosen groove positioning. One can note also that curves 2 and 3 in Fig. 3 corresponding different groove depth coincide practically.

B. Rectangular Grooves

Practical sapphire machining leads to a groove cross-section that reminds rather a trapezium than a wedge when the groove is narrow enough. A limit case of a trapezium is a rectangle. It is one of reasons why the most of our simulations were performed for the "rectangular" grooves.

Unlike the preceding issue the groove now is centered with respect to a current position namely of a WGH_{n11} regular mode radial maximum. It means D_g has a different value for different azimuthal indexes n and D_g corresponds D_g of WGH_8 mode, for example, if WGE_8 one is under analysis.

The first set of plots in Figs. 4, 5 represents the SWGR having format $\Phi = 2$, a diameter $D = 46.4$ mm, a height $h = 11.6$mm. The plots in Figs. 4, 5 are drawn for a constant groove width $w_g = 3$ mm.

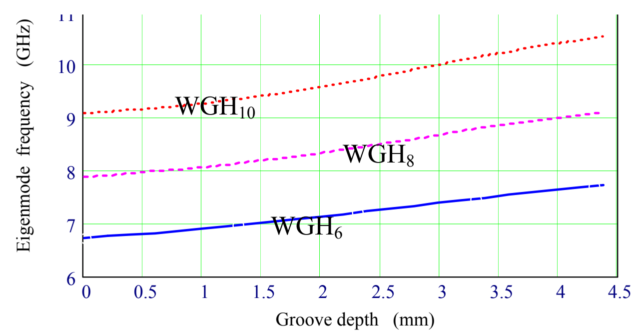


Fig. 4. Eigenmode frequencies vs. groove depth plots for WGH modes. $\Phi = 2$; groove width $w_g = 3$ mm; $T = 300$K.

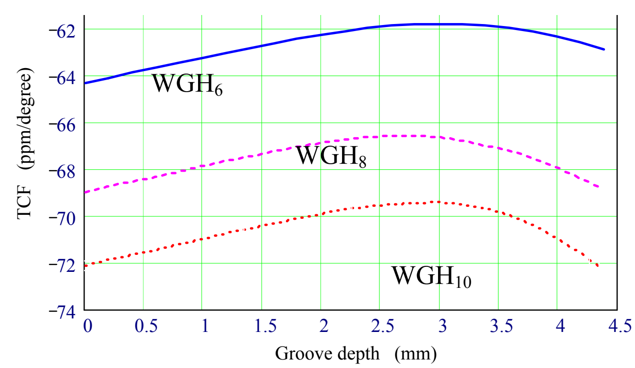


Fig. 5. TCF vs. groove depth plots for WGH modes. $\Phi = 2$; groove width $w_g = 3$ mm; $T = 300$K.

The resonant frequencies in Fig. 4 grow monotonically in parallel with h_g increase reflecting partial loss of the resonator volume. On the contrary, the TCF plots in Fig. 5 are not monotone. The TCF swings are in a range 2.4…2.9 ppm/deg.

Fig. 6 demonstrates a dependence of TCF vs. h_g plots in the case of n = 8 on mode polarization. The WGE-mode keeps its TCF almost constant. This is again a consequence of the groove positioning chosen in the issue that leads the WGE-mode has maximal intensity outside grooves (Fig. 7).

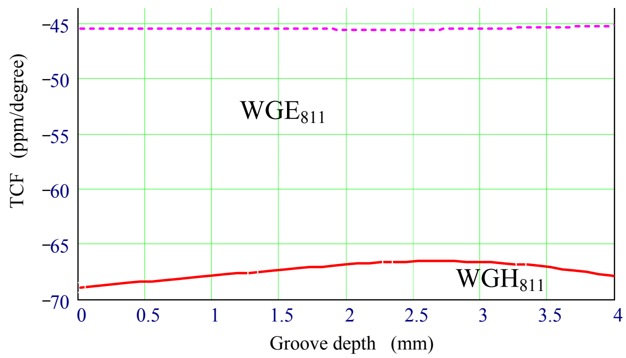


Fig. 6. TCF vs. groove depth plots for WGH$_{811}$ and WGE$_{811}$ modes. Φ = 2; groove width w_g = 3 mm; T = 300K.

a)

b)

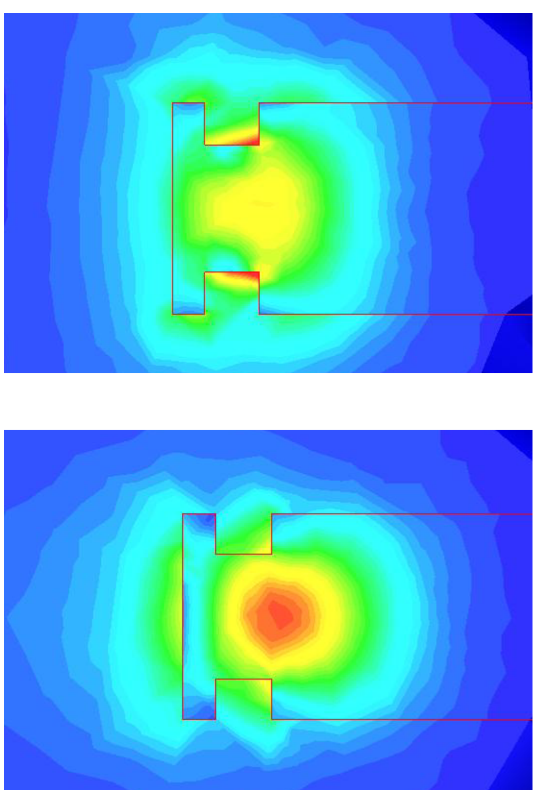


Fig. 7. Electric field distribution in z-r plane for WGH$_{811}$ (a) and WGE$_{811}$ (b) modes.

SWGR's characteristics, as known, depend on disk format. This subject is illustrated by Figs. 8, 9 describing the SWGR having Φ = 5, D = 50 mm, h = 5 mm. All plots are drawn for the same groove width w_g = 3 mm as before.

The TCF vs. h_g curves repeat basically that ones in Fig. 4. Both curves have equal TCF swings of 2.16 ppm/deg. Absolute TCF values are ~50% less in comparison with the previous case in accordance with the general tendency that is true in respect of WGE-modes as well.

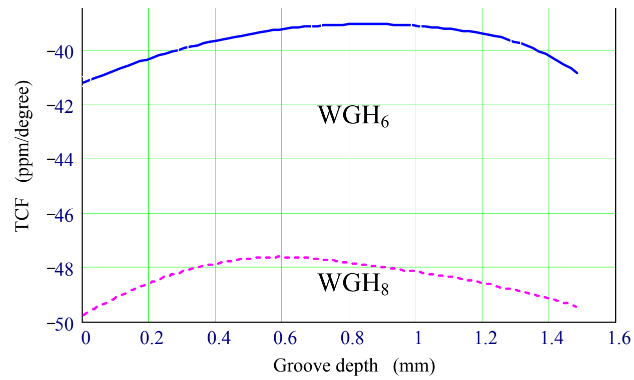


Fig. 8. TCF vs. groove depth plots for WGH-modes. Φ = 5; groove width w_g = 3 mm; T = 300K.

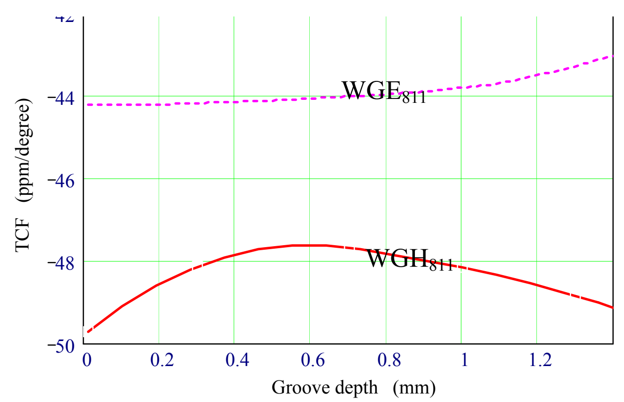


Fig. 9. TCF vs. groove depth plots for WGH$_{811}$ and WGE$_{811}$ modes. Φ = 5; groove width w_g = 3 mm; T = 300K.

More disk format does not prevent mode polarization to influence strongly TCF vs. h_g plots in Fig. 9 just in the same way as it was marked in Fig. 6. One can note a small TCF change of the WGE-mode in comparison with that of the WGH-mode.

The last set of plots in Figs. 10-12 reveals a role of the groove width. The groove height is fixed: h_g = 2.3 mm for Φ = 2 and h_g = 1 mm if Φ = 5.

Fig. 10 shows how a form of the TCF vs. w_g plots changes within used ranges of the parameters from a monotonic function to that one having an obvious extremum.

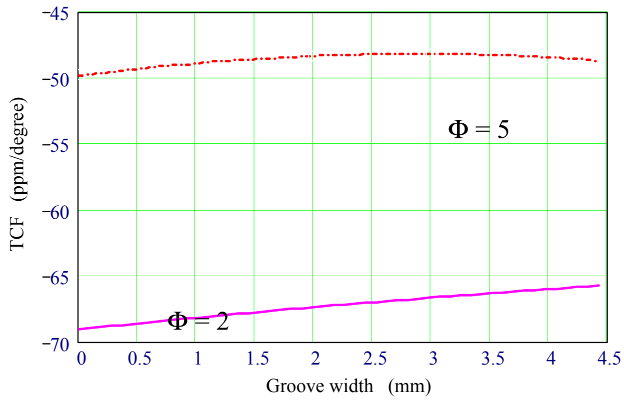


Fig. 10. TCF vs. groove width plots for WGH_{811} mode and two different disk formats. $h_g = 2.3$ mm for $\Phi = 2$ and $h_g = 1$ mm for $\Phi = 5$; $T = 300$K.

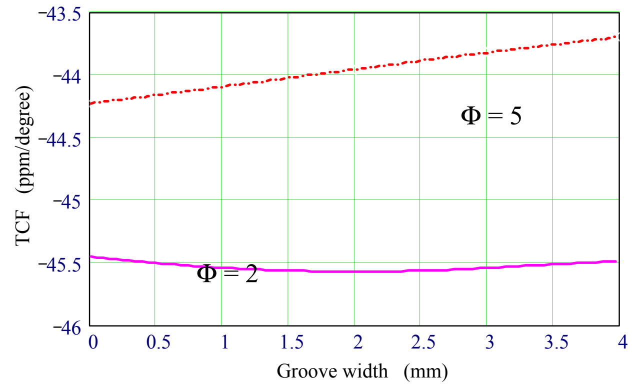


Fig. 12. TCF vs. groove width plots for WGE_{811} mode and two different disk formats. $h_g = 2.3$ mm for $\Phi = 2$ and $h_g = 1$ mm for $\Phi = 5$; $T = 300$K.

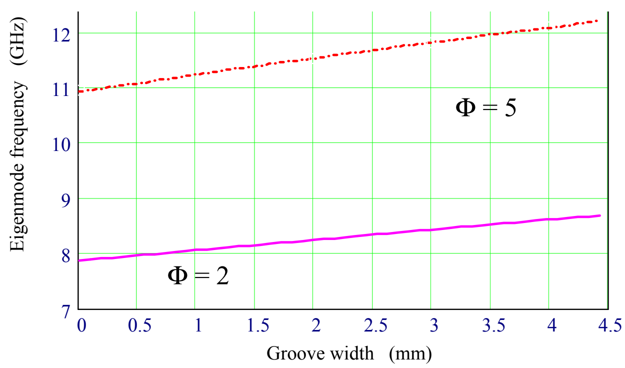


Fig. 11. Resonant frequency vs. groove width plots for WGH_{811} mode and two different disk formats. $h_g = 2.3$ mm for $\Phi = 2$ and $h_g = 1$ mm for $\Phi = 5$; $T = 300$K.

TCF variations do not exceed 3.5-5%. Thus, the resonant frequency plots in Fig. 11 look like straight lines.

The WGE_{811} mode (Fig. 12) demonstrates expected rather weak TCF variations with the w_g change.

IV. Conclusion

The double-sided grooves made on butt-ends of a sapphire disk in vicinity of an electric field maximum modify SWGR's resonant frequencies and TCF values. This effect, in principle, does not depend on a particular groove cross-section form. The wedge-shaped (V-type) and rectangular grooves used in our simulations gave close results.

The grooves influence predominantly the mode which electric field maximum radial position coincides with the groove radius. The TCF variation can reach here 5-10%.

The results of this investigation can be used for thin tuning of SWGR parameters as far as TCF and resonant frequencies at WGR-modes of different polarization is concerned.

Acknowledgment

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